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NMRCAL

I. USE OF NMRCAL

A. Introduction

NMRCAL is a program for calculating theoretical nmr spectra of 2 to 6 spin systems of spin-1/2 nuclei utilizing the usual LCAO-MO approach.[1,2] The program asks for the chemical shifts and coupling constants of the molecule, calculates the transitions, and displays them on the scope in stick figure form. A Lorentzian line shape of any desired line width can then be superimposed on the calculated transitions.

The program is useful as both a research and educational tool. It can be used to train students in the intricacies of the interpretation of nmr spectra, and it can be used to match experimental data. The spectra are stored in memory in a format identical to that used by the data acquisition system and the two displays can thus be readily compared. Since individual parameters such as a chemical shift or coupling constant can be quickly modified, the program enables the research nmr spectroscopist to match experimental data rapidly, and thus analyze the nmr spectrum without resort to card punching and large computer operations. Should the user desire to carry out iteration on a larger computer, the transitions and intensities can be punched out on paper tape for direct use on other computer systems. The calculated data can also be plotted out on an x-y plotter in both the stick form and the Lorentzian form for more detailed examination, using a sophisticated autoslew plot routine.

B. Hardware Requirements

An 8K NIC-1082 with hardware multiply-divide is required to calculate and plot spectra. If a 1083 (12K) is available, a listing and punched tape of the transitions and intensities can be produced on the Teletype for iteration on a larger computer.

C. Instructions for Use

The binary tape is loaded using the standard Binary Loader. The program starts at location 0 and can therefore be started by pressing Stored Program Start on the 1080 console. The Teletype will immediately type out:

```
NMRCAL
NO. of SPINS =
```

Answer this with any number between 2 and 6.

Terminate this and all other numerical entries with a Return.

SPECTROMETER FREQ (MHZ) =

Answer this question with the actual frequency of the spectrometer in use if the chemical shifts are going to be entered on the tau or delta scales. If they are to be entered on the frequency scale, the value of this parameter is unimportant.

SWEEP WIDTH =

This parameter determines what portion of the calculated spectrum is to be displayed. Any decimal number is a legal answer. One can enter the number in any notation desired. For instance, for a 500 cycle sweep width, one can type:

```
 500
 500.
 5.0E2
 0.5E3
+500.00E0
```

and so forth. The E refers to the power of ten (not the power of e). Termination of such an entry is with the first non-legal character, such as space, Return, Line Feed, or any alphabetic letter other than E.

At any time during input of decimal numbers, one can correct a mistake by typing a Rubout character. The Teletype will echo with a backslash and the program will completely erase the entry. The number can then be re-entered from the beginning.

OFFSET =

This parameter is the number of Hz from TMS, arbitrarily taken as zero Hz. This represents the right end of the display. The left end of the display is the offset plus the sweep width.

One can also enter the sweep width and offset in ppm by terminating the number with a D (for delta) instead of a carriage return. Do not type both a D and a Return as this is interpreted as two termination characters, and the second one will be read by the next question.

CHEMICAL SHIFTS
V(1) =
V(2) = etc.

The chemical shifts may be entered in any convenient decimal format, and in delta (δ) or tau (τ) units as well as in Hz. To enter a shift on the delta scale, terminate the input with a D. To enter one on the tau scale, terminate the value with a T.

The relationship between the tau, delta and frequency scales is illustrated below for a 60 MHz spectrometer.

240	180	120	60	0	-60	Hz (frequency) scale
6	7	8	9	10	11τ	Tau scale
4	3	2	1	0 TMS	-1δ	Delta scale (or ppm)

COUPLING CONSTANTS
J(1,2) =
J(1,3) = etc.

The coupling constants may be entered only in Hz (on the frequency scale), since coupling constants are not affected by the spectrometer frequency. Zero values may be entered by simply typing Return.

SAVE TRANS?

The transitions are saved in the third 4K stack (the second data memory stack) for later listing if this question is answered Y. Typing any other character will produce an N. If Y is answered when an 8K system is used, a question mark will be typed.

If the transitions are not saved, the calculation will be somewhat faster, and the second data stack may be used to store a SWAPped copy of another program, such as the FFT. If the transitions are saved, the modification of sweep width and offset parameters will take place immediately, instead of requiring a complete recalculation.

The program will then begin to calculate the spectrum. Two- and 3-spin systems are calculated essentially instantaneously, 4 spin systems in about 10 seconds, and 5 and 6 spin systems in 35 seconds and 5 - 8 minutes respectively.

When calculation is complete, the program will type a Return and ring the Teletype bell. The stick figure representation of the transitions will be displayed on the scope. Neither the horizontal nor the vertical display scale switch has any effect on this display.

The following commands are then available at the Teletype. They are discussed in detail below.

L	calculate a Lorentzian line shape from the transitions
J	change one or more coupling constants
V	change one or more chemical shifts
O	change the offset
W	change the sweep width
G	recalculate the spectrum using the modified parameters entered

R	restart the program from the beginning
P	plot out the line shape or stick figure representation on an XY plotter
T	list the transitions and intensities

II. MODIFICATION OF ENTERED PARAMETERS

L - Lorentzian Line Shape

This command allows one to superimpose a line shape of variable width on the stick figure transitions. The computer will first type out

```
LINE WIDTH =
```

Answer with the desired line width in Hz. If 0 or Return is selected, the stick-figure display will return. Once the line width has been specified, the calculation of the curves will begin. The amount of time this calculation takes is dependent on the line width selected and the number of transitions in the spectrum, but typically will be between 1 and 30 seconds.

When the calculation is complete, the Teletype bell will ring, a carriage return will be typed on the Teletype, and the calculated line shape will be displayed on the scope. This display is affected by the vertical display scale switch, although not by the horizontal one. The data occupies the first 1K of data memory and may also be examined in hardware readout mode. If only a straight line appears, change the Vertical Display Scale knob until the spectrum appears.

While the Lorentzian line shape is being displayed, the above list of commands is also operative and will have the same effect with one exception: a Plot command given during the curve display will cause a plot of the curves, while during the stick display, the stick figures will be plotted. A new line width can be entered by simply typing L again.

J - Modifying the Coupling Constants

Each time J is typed, the program will echo J(. Enter the index numbers representing the two nuclei which are coupled, followed by the new value of the coupling constant. The resulting typeout will be as shown below. The underlined numbers are typed by the user.

```
J(
J(2,
J(2,3) =
J(2,3) = 1.56
```

Note that the J, 2, 3 and 1.56 are entered by the user, but that the punctuation is automatically inserted by the program. The spectrum is not recalculated following this entry, so that more than one parameter can be modified if desired. The recalculation begins when G (Go) is typed.

V - Modify the Chemical Shifts

Just as in the J case, when one types V, the program will echo V(. The user then enters the nucleus number, the program types)= and allows the user to enter the new chemical shift. As before it may be in tau, delta or Hz units.

W and O - Sweep Width and Offset

Typing W causes the program to print

```
SWEEP WIDTH =
```

and allow entry of a new value in Hz or ppm, and typing O causes the program to print

```
OFFSET =
```

and allow entry of a new value. If only width and offset are modified, and the transitions have been saved in the second 4K of data memory, the new display should appear virtually instantaneously when the G command is given.

G - Go Recalculate the Spectrum

When G is typed, the program checks to see if a shift or coupling constant has been modified. If one has, the spectrum is recalculated from scratch. If only sweep width or offset has been modified, and Y was answered to SAVE TRANS? the new display is constructed from the transitions saved in memory.

P - Plot Out the Current Display

When P is typed, the program immediately begins the calibration routine and types out L. L. A dot is displayed in the left center of the screen, representing the lower left corner of the displayed spectrum. The plotter should now be energized and the bias and gain adjusted so that the pen is properly positioned. Typing U produces the response U. R., and causes a dot to be displayed in the upper right hand corner of the spectrum. The pen should then be adjusted for this corner. The dot will return to left center when D is typed and this U, D, U, D sequence can be repeated until the plotter is properly adjusted.

The plot made is of the currently displayed spectrum, either the stick figure or the Lorentzian one. Be sure either the Plotter Realtime or Plotter Autoslew button is depressed before proceeding, since the plotter outputs are unclamped only under these conditions. These buttons have no other effect under software control: the plotted routine contains autoslew features which are used no matter which button is depressed. This is an analog plot. To interface this routine to digitally driven recorders, please contact the factory.

To actually begin the plot be sure that the dot is in the L. L. position and type P again. The plotting rate is controlled by memory cycle timing, and can be increased by typing F and decreased by typing S. To discontinue plotting at once, type Q.

When the plot is completed, the program will type

TURN OFF PLOTTER.

The program will continue to display the last dot of the plotted spectrum until Return is typed. This gives one the opportunity to disable the plotter and prevent the pen from oscillating wildly when the display returns to the scope. If the plot has been much too fast, S may be typed several times here to slow the plot rate for the next plot attempt.

T - Type Out the Transitions and Intensities

The command T will cause the listing of the entire set of transitions and intensities on the Teletype. If the punch is turned on, the listing will be reproduced on paper tape as well. This listing can be interrupted at any time by typing Q.

R - Restart the Program

This command is entirely equivalent to pressing the Stored Program Start button. It causes a complete restart of NMRCAL at location 0.

TABLE I

Summary of NMRCAL Commands

```
NMRCAL
NO. OF SPINS =            (2 ≤ N ≤ 6)
SPECTROMETER FREQ. (MHZ) =
SWEEP WIDTH =             (Hz or D)
OFFSET =                  (Hz or D)

CHEMICAL SHIFTS
V(1) =                    (Hz, T or D)
V(2) =
  .
  .
  .
COUPLING CONSTANTS
J(1,2) =                  (Hz only)
  .
  .
  .
SAVE TRANS?               (Y or N)
```

Commands

```
J      change coupling constant
V      change chemical shift
O      change offset
W      change sweep width
G      go, recalculate with modified parameters
R      restart program
T      list transitions and intensities if saved
L      Lorentzian Line shape calculation
       LINE WIDTH =     (Hz only)
P      plot
       U    U.R.        upper right
       D    L.L.        lower left
       P    begins plotting
            F     faster
            S     slower
            Q     quit at once
       TURN OFF PLOTTER
            Return   -  to return to display
            F        -  faster next plot
            S        -  slower next plot
```

III. METHOD OF CALCULATION

A. Hand Calculation Methods

In order to calculate nmr spectra, it is necessary to consider all possible spin states that the spin system might assume. We will represent the nuclear spin $I_z = +1/2$ by α and $I_z = -1/2$ by β. For a two spin system consisting of nuclei numbered 1 and 2, there are four possible arrangements of the spins:

$$\begin{array}{ccc} & \alpha(1)\,\beta(2) & \\ \alpha(1)\,\alpha(2) & & \beta(1)\,\beta(2) \\ & \beta(1)\,\alpha(2) & \end{array}$$

These are basic wave functions or basis functions, describing the spin states.

We will introduce the abbreviation here that the order in which the spin states are written indicates the number of the nucleus referred to. Thus $\alpha(1)\beta(2)$ will be abbreviated $\alpha\beta$, and $\beta(1)\alpha(2)$ will become $\beta\alpha$.

However, according to quantum mechanics, it is not permissible to label particular nuclei as to spin any more than we can pick out a single electron from a molecule and number it. A legitimate wave function must not change if the spins within it are interchanged. Therefore, while it is legal to say that both nuclei have the same spin by writing $\alpha\alpha$ or $\beta\beta$, we cannot say that nucleus number 1 has spin α and nucleus 2 spin β.

Instead, we must "mix" the two wave functions $\alpha\beta$ and $\beta\alpha$ using an LCAO-MO approach and arrive at two new wave functions in which we do not violate the Uncertainty Principle. It is in the calculation of these mixed wave functions, and their corresponding energy levels and transitions, that the computer becomes an invaluable tool. The general form of the mixed wave functions is illustrated below:

$$\psi_2 = C_1\alpha\beta + C_2\beta\alpha$$

$$\psi_3 = C_3\alpha\beta + C_4\beta\alpha$$

We can solve for the coefficients $c_1 \ldots c_4$ by some simple determinant method, in this case even by hand by utilizing the usual requirements of quantum mechanics that wave functions be normalized:

$$\int \psi^2 d\tau = 1$$

and orthogonal

$$\int \psi_a \psi_b d\tau = 0 \quad \text{for } a \neq b$$

where $d\tau$ is used to indicate the integral over all space.

For the two spin case illustrated here it can easily be shown[3] that

$$\psi_2 = \frac{1}{\sqrt{2}}\alpha\beta + \frac{1}{\sqrt{2}}\beta\alpha \text{ and } \psi_3 = \frac{1}{\sqrt{2}}\alpha\beta - \frac{1}{\sqrt{2}}\beta\alpha$$

The actual energies can then be calculated using a form of the Schroedinger wave equation

$$H\psi = E\psi$$

The transitions between these energy levels are then found utilizing the products of the coefficients.

The problem becomes more complex as the spin systems become larger, however. For a three spin case, for example, there are many more basis functions:

$$\begin{array}{cccc} & \alpha\alpha\beta & \alpha\beta\beta & \\ \alpha\alpha\alpha & \alpha\beta\alpha & \beta\alpha\beta & \beta\beta\beta \\ & \beta\alpha\alpha & \beta\beta\alpha & \end{array}$$

and the number of basis functions increases with spin number according to Pascal's triangle:

Spins	
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1

In order to handle these increasingly complex systems effectively, we perform the calculations in a somewhat different order within the computer utilizing matrix algebra.

B. General Description of the Computer Method

Within the computer, we prepare lists of the spin states, grouped according to the total spins of the basis function. In other words, for each group of basis functions, the quantity F_z is the same, where

$$F_z = \sum_{i=1}^{n} I_z(i)$$

and $I_z(i) = +1/2$ or $-1/2$ as before.

For each group of basis functions we construct a matrix, called the H-matrix, utilizing the equations given in the next section and then diagonalize it. By diagonalize, we mean that we add and subtract elements in rows and columns until all off-diagonal elements are zero and non-zero values remain only in the diagonal.[4] These diagonal elements are the energy levels associated with that set of mixed wave functions.

During the process of diagonalization, we keep track of the numbers added and subtracted from the H-matrix in a second matrix called the U-matrix. When diagonalization is complete, the columns of the U-matrix contain the coefficients of the mixed wave functions.

According to theory,[1-3] transitions can take place only between energy levels having an F_z differing by 1. Since each set of basis functions has one more spin in the "up" state than the last, this means that transitions occur only between energy levels produced by adjacent sets of basis functions. It is further required that the basis functions associated with the energy levels must differ only in the interchange of one spin. The probability of these transitions is given by the product of the coefficients of the wave functions as described below.

In summary, then, the basis functions are sorted into groups according to their F_z, a matrix is constructed from them and then diagonalized. This data is then saved, while the next matrix is constructed and diagonalized. Transitions occur only between these adjacent states and these are calculated and stored for display. The latest matrices and energy levels are then moved to the old matrix position and a new matrix constructed from the next set of basis functions. The process is repeated until all groups of basis functions have been used.

C. Mathematics of the Computer Method

For each F_z, the basis functions are listed internally and counted. The H-matrix is a square matrix having dimensions equal to the number of basis functions having that F_z.

The diagonal elements are constructed according to the equation

$$H_{uu} = \sum_{i=1}^{n} \left[\nu_i I_z(i) + \sum_{j=i}^{n} J_{ij} T_{ij} \right]$$

where ν_i is the chemical shift of nucleus i

$I_z(i) = +1/2$ for spin α and $-1/2$ for spin β

J_{ij} is the coupling constant between nuclei i and j

$T_{ij} = +1/4$ if spins i and j of the basic function corresponding to that diagonal element are aligned and $-1/4$ if they are opposed.

For example, using the list of basis functions

$\alpha\alpha\alpha\beta$	H_{11}
$\alpha\alpha\beta\alpha$	H_{22}
$\alpha\beta\alpha\alpha$	H_{33}
$\beta\alpha\alpha\alpha$	H_{44}

we construct H_{22} of a 4 × 4 matrix using the basis function $\alpha\alpha\beta\alpha$

$$H_{22} = \nu_1 + \nu_2 - \nu_3 + \nu_4 + J_{12} - J_{13} + J_{14} - J_{23} + J_{24} - J_{34}$$

The off-diagonal elements are calculated by

$$H_{uv} = 1/2 J_{ij} U$$

where $U = 1$ if and only if basis function u and basis function v differ only in the interchange of spins i and j
and $U = 0$ otherwise
and J_{ij} is the coupling constant between the two spins which differ in the two basis functions.

For example, using the same set of basis functions as above, $H_{1,2}$ is constructed utilizing $\alpha\alpha\alpha\beta$ and $\alpha\alpha\beta\alpha$ which differ in the interchange of spins 3 and 4.

Then $H_{1,2} = 1/2\ J_{3,4}$

Similarly $H_{2,4} = 1/2\ J_{1,3}$

The matrix is then diagonalized using the Jacobian method.[5] The energy levels or eigenvalues of the four mixed wave functions are found in the diagonal elements of the H-matrix. The coefficients or eigenvectors of the wave functions are found in the columns of the U-matrix:

$$\left|\begin{array}{cccc} E_1 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 \\ 0 & 0 & E_3 & 0 \\ 0 & 0 & 0 & E_4 \end{array}\right| \qquad \begin{array}{c} \alpha\alpha\alpha\beta \\ \alpha\alpha\beta\alpha \\ \alpha\beta\alpha\alpha \\ \beta\alpha\alpha\alpha \end{array} \qquad \begin{array}{cccc} \psi_1 & \psi_2 & \psi_3 & \psi_4 \\ \left|\begin{array}{c} C_{11} \\ C_{21} \\ C_{31} \\ C_{41} \end{array}\right. & \begin{array}{c} C_{12} \\ C_{22} \\ C_{32} \\ C_{42} \end{array} & \begin{array}{c} C_{13} \\ C_{23} \\ C_{33} \\ C_{43} \end{array} & \left.\begin{array}{c} C_{14} \\ C_{24} \\ C_{34} \\ C_{44} \end{array}\right| \end{array}$$

H-Matrix Basis Functions U-Matrix

where C_{11}, C_{21}, C_{31} and C_{41} are the coefficients of ψ_1, etc.

The energies of the transitions, in hz, are calculated by subtracting each energy level in the previous H-matrix diagonal from each energy level in the current H-matrix diagonal. The probability of the intensity of these transitions is given by

$$I_{km} = \left(\sum_{i=1}^{n} \sum_{j=1}^{n} C_{ik} C'_{jm} X_{ij} \right)^2$$

where
I_{km} is the intensity of the transition between energy level k of the last H-matrix and energy level m of this H-matrix.
C_{ik} is the i^{th} coefficient of the k^{th} column of the last U-matrix
C'_{jm} is the j^{th} coefficient of the m^{th} column of this U-matrix
X_{ij} = 1 if and only if basis function k of the last set and basis function m of the current set differ only in one spin
= 0 in all other cases.

These transitions are, in each case, converted to an integer representing an address in a 1024 point array, their intensities scaled to fit in a 12-bit display and added to any previous transitions stored in the array. Thus, the stick figure array consists of 1024 bins, into which all transitions within the specified sweep width and offset must be fit. If the transitions are to be saved, they are stored in floating point format in the second data stack.

The Lorentzian line shape calculation is carried out by calling for the line width at half height. The spin-spin relaxation time T_2 can be found by

$$\text{line width} = 1/\pi T_2$$

Each point on the Lorentzian curve is calculated by

$$g(\nu) = \frac{g_{max}}{1 + [2\pi T_2 (\nu - \nu_0)]^2}$$

where g_{max} is the maximum amplitude of the line
$g(\nu)$ is the amplitude at frequency ν
$\nu - \nu_0$ = the distance in hz between the resonance frequency and the current point on the curve.

TABLE II

STORAGE MAP FOR DATA USED IN NMRCAL

0 - 4312	Program Storage
6000 - 7577	Floating Point Package
100000 - 100643	H-Matrix in packed upper diagonal format
100645 - 102304	Current eigenvectors matrix (U-matrix)
102305 - 103744	Previous eigenvectors matrix
103745 - 104014	List of current energy levels (eigenvalues)
104015 - 104065	List of previous energy levels
104215 - 104241	Current basis functions
104242 - 104264	Previous basis functions
104265 - 104301	Chemical shifts
104302 - 104354	Coupling constants
106000 - 107777	Stick display array
100000 - 101777	Lorentzian array (overwrites H and U matrices)
110000 - 111000	Transitions and intensities in floating point format

BIBLIOGRAPHY

1. Pople, J. A., Schneider, W. G. and Bernstein, H. J. High Resolution Nuclear Magnetic Resonance, McGraw-Hill, 1959.

2. Becker, E. D., High Resolution NMR, Academic Press, 1969.

3. Roberts, J. D., An Introduction to the Analysis of Spin-Spin Splitting in High Resolution Nuclear Magnetic Resonance Spectra, W. A. Benjamin, 1962.

4. Bevington, P. R., Data Reduction and Error Analysis for the Physical Sciences, McGraw-Hill, 1969.

5. Ralston, A. and Wilf, A., Mathematical Methods for Digital Computers, Vol. I, John Wiley & Sons, 1960.